ENEE 610
Th 10/15/20

Note Title 9/19/2020

4 Canonical LPR forms
VLSI page

Semistate for polynomial transfer functions
Hysteresis using op-amps
Convolution

$$f(s) = \frac{f(s)+f(-s)}{2} + \frac{f(s)-f(-s)}{2} = Ev[f(s)] + Od[f(s)]$$

LPR's are odd

1st Foster, 2nd Foster } partial fractions

1st Cauer, 2nd Cauer } continued fractions @ ∞, @ 0

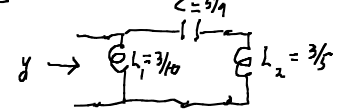
$$y(s) = \frac{5(s^2+2)}{s(s^2+3)} = \frac{5s^2+10}{s^3+3s}$$

for 2nd Cauer

$$3s+s^3 \,\big)\, 10+5s^2 \quad \to \frac{10}{3s}$$
$$10 + \frac{10}{3}s^2$$
$$\frac{5}{3}s^2 \,\big)\, 3s+s^3 \quad \to \frac{9}{5s}$$
$$3s$$
$$s^3 \,\big)\, \frac{5}{3}s^2 \quad \to \frac{5}{3s}$$

$$\Rightarrow y(s) = \frac{10}{3s} + \cfrac{1}{\cfrac{9}{5s} + \cfrac{1}{5/3s}}$$

y, z, y: $C = 5/9$



2nd Cauer

for 1st Cauer

$$5s^2+10 \,\big)\, s^3+3s \quad \to \frac{1}{5}s \;(y)$$
$$s^3+2s$$
$$s \,\big)\, 5s^2+10 \quad \to 5s \;(z)$$
$$5s^2$$
$$10 \,\big)\, s \quad \to \frac{1}{10}s \;(y)$$
$$s$$
$$0 = 1/\infty \;(y)$$

1st Cauer

$$E\dot{x} = Ax + Bu$$
$$y = Cx$$
$$y = C(sE - A)^{-1}Bu$$

$$y = [c_1\ c_2]\begin{bmatrix} 0 & -1 \\ -1 & -s \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}u$$
$$= [c_1\ c_2]\begin{bmatrix} -b_2 \\ -b_1 - s b_2 \end{bmatrix}u = -b_2 c_1 - b_1 c_2 - s b_2 c_2$$

if $y(s) = s \Rightarrow b_1 = c_1 = 0,\ b_2 = -1,\ c_2 = 1$

$$y(s) := \begin{pmatrix} c1 & c2 & c3 & c4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & -s \\ -s & -1 & 0 & -s^2 \\ -s^2 & -s & -1 & -s^3 \end{pmatrix} \cdot \begin{pmatrix} b1 \\ b2 \\ b3 \\ b4 \end{pmatrix}$$

matrices for poles at infinity, n=1,2,3 can continue by induction

Es-A

$$\begin{pmatrix} s & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

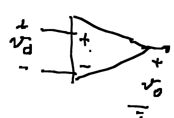
inverse of Es-A

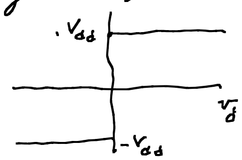
$$\begin{pmatrix} 0 & -1 \\ -1 & -s \end{pmatrix}$$

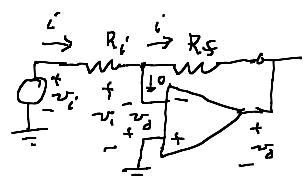
$$\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & -s \\ -s & -1 & -s^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & -s \\ -s & -1 & 0 & -s^2 \\ -s^2 & -s & -1 & -s^3 \end{pmatrix}$$

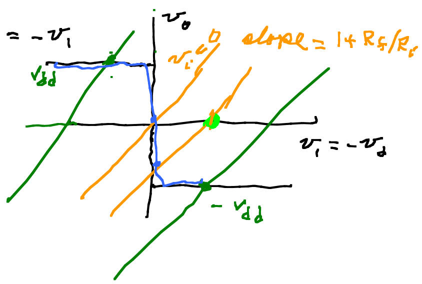
op-amp amplifies v_o







as $v_d = -v_1$



$v_i = R_i i + v_1$ $\Rightarrow i = \dfrac{v_i - v_1}{R_i}$

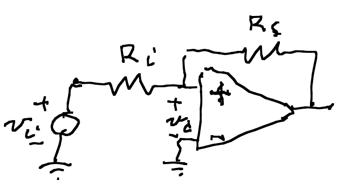
$v_1 = R_f i + v_0$

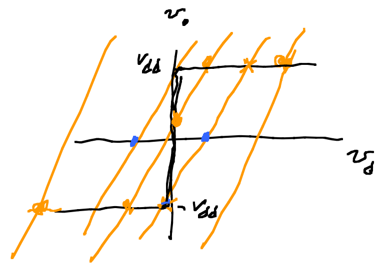
$= \dfrac{R_f}{R_i}(v_i - v_1) + v_0 \Rightarrow v_0 = (1 + \dfrac{R_f}{R_i})v_1 - \dfrac{R_f}{R_i} v_i$

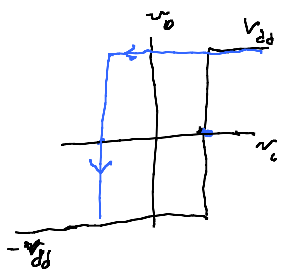
if $v_i \neq 0$ v_1 intercept is $v_1 = \dfrac{R_f/R_i}{1 + \frac{R_f}{R_i}} v_i =$ point

if $v_1 = 0$ then $v_i = R_i i$; $v_0 = -R_f i$

$$\frac{v_0}{v_i} = \frac{-R_f}{R_i}$$







Convolution: $g(t) * h(t) = \displaystyle\int_{-\infty}^{\infty} g(t-\tau)\, h(\tau)\, d\tau \Leftarrow$ definition

If $g(t) = \delta(t)$ [unit area, $\delta(t)$] so $\displaystyle\int_{-\infty}^{t} \delta(\tau)\, d\tau = 1(t)$

$$g(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(t-\tau) h(\tau)\, d\tau = h(t) \int_{-\infty}^{\infty} \delta(t-\tau)\, d\tau$$

$$= h(t) \cdot 1 = h(t)$$

$\mathcal{L}[g * h] = \mathcal{L}[g] \cdot \mathcal{L}[h]$;

$$\int_{-\infty}^{\infty} g(t) * h(t) e^{-at} dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t-\tau) h(\tau) d\tau e^{-at} dt = \iint g(t-\tau) h(\tau) e^{-at} dt d\tau$$

Let $t - \tau = x$, $dx = \cancel{t} - d\tau$ for a fixed t, $dx = dt - \cancel{\tau}$ for fixed τ

$t = x + \tau$
$$\int_{-\infty}^{\infty}\int^{\infty} g(x) h(\tau) e^{-a(x+\tau)} dx d\tau = \int g(x) e^{-ax} dx \cdot \int h(\tau) e^{-a\tau} d\tau$$
$$= \mathcal{L}[g] \cdot \mathcal{L}[h]$$

$$\mathcal{L}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-at} dt = e^{-a \cdot 0} \cdot \int_{-\infty}^{\infty} \delta(t) dt = e^{-0} = 1$$